RISK ANALYSIS IN CAPITAL BUDGETING

Risk is the chance that something unfavourable event might occur.
Capital budgeting decisions takes place in a dynamic and uncertain environment, therefore there is uncertainity associated with the future cashflows of the project.
There are normally 2 types of risks facing the company.

i) Financial risks
ii) Business / total / operating risks

i) Financial risks

They are risks associated with the use of debt in the capital structure. Therefore risks will be measured using gearing ratio.

ii) Business Risks / Total risks

They are risk associated with declining of the company's earnings. Its known as total risks because it is a combination of:
- Systematic risk
- Unsystematic risks.

- **Systematic risks**

This is a risk affecting all the firms in the industry and hence they cannot be minimized by holding a portfolio. eg Inflation, political risks, government policy, energy crisis.

- **Unsystematic risks**

This is a risk affecting a specific company or industry alone. Therefore, they can be minimized by holding a portfolio eg labour dispute, strikes, changes in market share value.

Methods of evaluating Risks

1. Payback period
2. Expected monetary value (EMV) / Expected value (EV)
3. Standard deviation (σ) / Variance (σ^2)
4. Coefficient of Variation (CV)

1. Expected Monetary value (EMV)

This is similar to arithmetic mean/average of the projects cashflows. Its calculated as follows.

$$EMV = \frac{\Sigma \text{ cashflows}}{n}$$ ----- In absence of probability

$$EMV = \Sigma (\text{cashflows} \times \text{prob})$$ --- In presence of prob

The higher the EMV, the better the project.

2. Standard Deviation (δ)

This is the absolute measure of risk and it is used to determine the degree of which the actuals are different from the expected.

→ the lower the δ, the less risky the project is and vice versa.

→ Its computed as follows depending with the nature of information.

$$\sigma = \sqrt{\Sigma (R - \Sigma R)^2 \text{prob}}$$ ---- with presence of prob.

$$\sigma = \sqrt{\frac{\Sigma (R - \Sigma R)^2}{n-1}}$$ ---- In absence of prob and sample size is less than 30 ($n < 30$)

$$\sigma = \sqrt{\frac{\Sigma (R - \Sigma R)^2}{n}}$$ ---- In absence of prob and sample size is greater than 30 ie ($n > 30$)

where = R → Return.
ΣR → Expected Return / EMV.

If you square standard deviation, you get variance. ie
δ^2 = variance.

3 Coefficient of Variation (CV).

This is a relative measure of risk. It is used to evaluate mutually exclusive projects if the EMV and δ gives conflicting results

→ It shows the risk per unit of return.

→ the lower the CV, the less risky the project is.

→ Its calculated as follows:

$$CV = \frac{\delta}{EMV}$$

NOV 2015 Q 5 d)

$EMV = 40 \times 0.2 + 15 \times 0.6 + -10 \times 0.2 = 15\%$

$\sigma = \sqrt{\Sigma (R - \Sigma R)^2 \text{prob}}$

$(40 - 15)^2\ 0.2 = 125$

$(15 - 15)^2\ 0.6 = 0$

$(-10 - 15)^2\ 0.2 = 125$

$\sqrt{250} = \underline{15.81}$

$$CV = \frac{\delta}{EMV} = \frac{15.81}{15} = \underline{1.054}$$

Scanned with CamScanner

Nov 2011 Q 2d.

An investor has 2 securities, A and B with the following returns characteristics:

State of Economy	prob	Returns (A) %	Returns Security B %
Recession.	0.3	12	6
Stable	0.4	15	7.5
Expansion	0.3	10	5

Required

Assess the riskiness of securities A and B (5 mks)

Solution:

i) EMV = Σ (cashflows x prob)

$A = 12 \times 0.3 + 15 \times 0.4 + 10 \times 0.3 = 12.6$

$B = 6 \times 0.3 + 7.5 \times 0.4 + 5 \times 0.3 = 6.3$

ii) Standard deviation

$$\sigma = \sqrt{\Sigma (R - ER)^2 \text{prob}}$$

σA

$(12 - 12.6)^2\ 0.3 = 0.108$

$(15 - 12.6)^2\ 0.4 = 2.304$

$(10 - 12.6)^2\ 0.3 = 2.028$

$\sqrt{4.437}$

$\sigma = 2.106$

σB

$(6 - 6.3)^2\ 0.3 = 0.027$

$(7.5 - 6.3)^2\ 0.4 = 0.576$

$(5 - 6.3)^2\ 0.3 = 0.507$

$\sqrt{1.11}$

$\sigma = 1.05$

ii) $CV = \frac{\sigma}{EMV}$

$CVA = \frac{2.106}{12.6} = 0.167$

$CVB = \frac{1.05}{6.3} = 0.167$

June 2013 Q 3a

Solution

i) Total expected return.

$$\% \text{ return} = \frac{(P_1 - P_0) + \text{Dividend income}}{P_0} \times 100\%$$

where $P_1 - P_0$ = Capital gain.

$X = \frac{(34 - 30) + 3.4}{30} \times 100\% = 24.67\%$

$Y = \frac{(69 - 72) + 4.7}{72} \times 100\% = 2.36\%$

$Z = \frac{(146 - 140) + 4.8}{140} \times 100\% = 7.7\%$

ii) Relative returns (RR)

RR = Capital gain + Dividend income

$X = 4 + 3.4 = 7.4$

$Y = -3 + 4.7 = 1.7$

$Z = 6 + 4.8 = 10.8$

iii) Expected Return of portfolio

$ERP = W_X ER_X + W_Y ER_Y + W_Z ER_Z$

where $W \rightarrow$ weight / proportion

$ERP = \frac{4}{10} \times 24.67 + \frac{2}{10} \times 2.36 + \frac{4}{10} \times 7.7 = \underline{13.44\%}$

May 2012 Q 5d.

West Ltd has forecasted the following end of period prices for its shares

End of period price per share (sh)	probability
35	0.15
42	0.10
50	0.30
55	0.20
60	0.25

The current price per share is sh 50

Required.

i) Expected Returns

ii) Variance of end of period returns

Solution:

i) Expected Return

$\% \text{ Return} = \frac{P_1 - P_0}{P_0} \times 100\%$ where P_1 - Expected price (End period price, P_0 - Current price

$(35 - 50) \div 50 = -30\%$

$(42 - 50) \div 50 = -16\%$

$(50 - 50) \div 50 = 0$

$(55 - 50) \div 50 = 10\%$

$(60 - 50 \div 50 = 20\%$

$ER = -30 \times 0.15 + -16 \times 0.1 + 0 \times 0.3 + 10 \times 0.2 + 20 \times 0.25 = \underline{0.9}$

(ii) Variance (σ^2)

$\sigma = \sqrt{\Sigma (R - ER)^2} \text{ prob}$

$(-30 - 0.9)^2 \ 0.15 = 109.35$

$(-16 - 0.9)^2 \ 0.10 = 28.561$

$(0 - 0.9)^2 \ 0.30 = 0.243$

$(10 - 0.9)^2 \ 0.20 = 16.562$

$(20 - 0.9)^2 \ 0.25 = 91.2025$

Variance. $\underline{245.9185}$

Assignment: Nov 2019 Q3b
June 2011 Q1
Aug 2009 Q4b
Dec 2013 Q5d.
Nov 2018 Q5c

NOV 2018 Q 5C

Solution

(i) Expected Return (EMV)

A = 12×0.3 + 15×0.4 + 10×0.3 = 12.6.

B = 6×0.3 + 7.5×0.4 + 5×0.3 = 6.3

(ii) Standard deviation (σ)

$$\sigma = \sqrt{\Sigma (R - ER)^2 \text{ prob}}$$

σA

$(12 - 12.6)^2\ 0.3 = 0.108$

$(15 - 12.6)^2\ 0.4 = 2.304$

$(10 - 12.6)^2\ 0.3 = 2.025$

$\sqrt{4.437}$

$\sigma = 2.106$

σB

$(6 - 6.3)^2\ 0.3 = 0.027$

$(7.5 - 6.3)^2\ 0.4 = 0.576$

$(5 - 6.3)^2\ 0.3 = 0.507$

$\sqrt{1.11}$

$\sigma = 1.05$

(iii) Correlation Coefficient of the two security returns

$$r_{AB} = \frac{COV_{xy}}{\sigma_A \sigma_B}$$

$COV_{xy} \rightarrow$ Covariance b/w x and y

$COV_{AB} = \Sigma (R_A - ER_A)(R_B - ER_B)$ prob

$(12 - 12.6)(6 - 6.3)\ 0.3 = 0.054$

$(15 - 12.6)(7.5 - 6.3)\ 0.4 = 1.152$

$(10 - 12.6)(5 - 6.3)\ 0.3 = 1.014$

2.193

$$r_{AB} = \frac{2.193}{2.106 \times 1.05} = \underline{+1}$$

There is a perfect positive correlation between security A & B returns

(iv) ~~Risk of the portfolio~~

(v) Expected Return of the portfolio

$ERP = W_A\, ER_A + W_B\, ER_B$

$0.4 \times 12.6 + 0.6 \times 6.3 = 8.82$

(v) Risk of the portfolio

$$\sigma p = \sqrt{W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2 W_A W_B\, COV_{AB}}$$

$$= \sqrt{0.4^2 \times 2.106^2 + 0.6^2 \times 1.05^2 + 2 \times 0.4 \times 0.6 \times 2.193}$$

$$\sigma p = \underline{1.47}$$

Scanned with CamScanner

NOV 2019 Q 4c

(i) $ERP = W_C ER_C + W_R ER_R$

$ER_C = 16 \times 0.2 + 12 \times 0.6 + 8 \times 0.2 = 12$

$ER_R = 14 \times 0.2 + 10 \times 0.6 + 6 \times 0.2 = 10$

~~ERP=~~ weights = Coral Ltd. = 300,000 ⇒ 0.6

Reef Ltd = 200,000 ⇒ 0.4

500,000

$ERP = 0.6 \times 12 + 0.4 \times 10 = \underline{11.2\%}$

(ii) Actual portfolio risk

$$\sigma p = \sqrt{W_x^2 \sigma x^2 + W_y^2 \sigma y^2 + 2 W_x W_y \, COV_{xy}}$$

$COV_{xy} = \Sigma (R_x - ER_x)(R_y - ER_y) \text{ prob}$

$(16-12)(14-10)\ 0.2 = 3.2$

$(12-12)(10-10)\ 0.6 = 0$

$(8-12)(6-10)\ 0.2 = 3.2$

$\underline{6.4}$

$\sigma = \sqrt{\Sigma (R - ER)^2 \text{ prob}}$

σC

$(16-12)^2\ 0.2 = 3.2$

$(12-12)^2\ 0.6 = 0$

$(8-12)^2\ 0.2 = 3.2$

$\sqrt{6.4}$

$\sigma = 2.53$

σR

$(14-10)^2\ 0.2 = 3.2$

$(10-10)^2\ 0.6 = 0$

$(6-10)^2\ 0.2 = 3.2$

$\sqrt{6.4}$

$\sigma = 2.53$

$$\sigma p = \sqrt{0.6^2 \times 2.53^2 + 0.4^2 \times 2.53^2 + 2 \times 0.6 \times 0.4 \times 6.4}$$

$\sigma p = \underline{2.53}$

Scanned with CamScanner

PORTFOLIO ANALYSIS

A portfolio in financial term refers to collection of investment opportunities or securities with an aim of minimizing the risk.

Measures of the relationship btw securities returns

There are normally 2 measures used:

1. Co-variance (COV)
2. Correlation Coefficient. (r)

1. Co-Variance (COV)

The COV between 2 securities indicates whether there is a positive or negative relationship but it does not give the extent.

→ Its calculated as follows:

$$COV_{xy} = \sum (R_x - ER_x)(R_y - ER_y) \text{ prob} \text{ --- presence of prob}$$

$$COV_{xy} = \frac{\sum (R_x - ER_x)(R_y - ER_y)}{n-1} \text{ --- Absence of prob.}$$

2. Correlation Coefficient (r)

This is a statistical method used to determine the nature and strength of the relationship existing between securities returns.

→ Its computed as follows:

$$r = \frac{COV_{xy}}{\sigma_x \sigma_y}$$

The result obtained using formular above is evaluated as follows

$\pm 0 - 0.29$ → Weak correlation

$\pm 0.3 - 0.69$ → Moderate correlation

$0.7 - 0.99$ → Strong correlation

± 1 → perfect correlation

$+$ → Direct relationship

$-$ → Indirect relationship

other evaluations

1. Expected Return of portfolio

$$ERP = W_x ER_x + W_y ER_y$$

where W → weight / proportion.

2. Weighted standard deviation of portfolio

$$\text{Weighted } \sigma_p = W_x \sigma_x + W_y \sigma_y$$

3 Actual portfolio risk

$$\sigma p = \sqrt{Wx^2\sigma x^2 + Wy^2\sigma y^2 + 2\,Wx\,Wy\,COVxy}$$

$rxy\,\sigma x\,\sigma y$.

where $COVxy$ is the same as $rxy\,\sigma x\,\sigma y$

4. Percentage of Risk diversification:

$$\%\text{ of risk diversification} = \frac{\text{Weighted }\sigma p - \text{Actual }\sigma p}{\text{Weighted }\sigma p} \times 100\%$$

Illustration

Consider the following 2 securities X and Y with the following ~~future~~ features:

probability	Return of X(%)	Returns of Y(%)
0·3	25	18
0·4	20	14
0·3	15	10

Required. (i) Expected return of each security
(ii) Expected Return of the portfolio assuming 60% to 40% b/w X & Y
(iii) Covariance b/w X and Y returns
(iv) Standard deviation of each security
(v) Correlation coefficient
(vi) ~~the~~ Weighted standard deviation of the portfolio
vii) Actual portfolio risk
v) % of Risk diversification.

Solution:

(i) Expected return

$ERx = 25\times0{\cdot}3 + 20\times0{\cdot}4 + 15\times0{\cdot}3 = 20$

$ERy = 18\times0{\cdot}3 + 14\times0{\cdot}4 + 10\times0{\cdot}3 = 14$

ii) ERP assuming weights of 60% and 40% for X and Y respectively

$ERP = 20\times0{\cdot}6 + 14\times0{\cdot}4 = 17{\cdot}6$.

(iii) Covariance COV

$COVxy = \Sigma(Rx - ERx)(Ry - ERy)\,prob$

$(25-20)(18-14)\,0{\cdot}3 = 6$

$(20-20)(14-14)\,0{\cdot}4 = 0$

$(15-20)(10-14)\,0{\cdot}3 = 6$

$COVxy\quad 12$

(iv) Standard deviation

$\sigma = \sqrt{\Sigma(R-ER)^2\,prob}$

σx

$(25-20)^2\,0{\cdot}3 = 7{\cdot}5$

$(20-20)^2\,0{\cdot}4 = 0$

$(15-20)^2\,0{\cdot}3 = 7{\cdot}5$

$\sqrt{15}$

$\sigma = 3{\cdot}873$

σy

$(18-14)^2\,0{\cdot}3 = 4{\cdot}8$

$(14-14)^2\,0{\cdot}4 = 0$

$(10-14)^2\,0{\cdot}3 = 4{\cdot}8$

$\sqrt{9{\cdot}6}$

$\sigma = 3{\cdot}098$

(V) Correlation Coefficient (r)

$$r = \frac{COV_{xy}}{\sigma x \sigma y} = \frac{12}{3.87 \times 3.098} = +1$$

This means there is a positive correlation between security X and Y returns.

(VI) Weighted standard deviation of portfolio

Weighted $\sigma p = W_x \sigma x + W_y \sigma y$

$$0.6 \times 3.87 + 0.4 \times 3.098 =$$
$$= \underline{\underline{3.56}}$$

(VII) Actual portfolio risk

$$\sigma p = \sqrt{W_x^2 \sigma x^2 + W_y^2 \sigma y^2 + 2 W_x W_y \, COV_{xy}}$$

$$= \sqrt{0.6^2 \times 3.87^2 + 0.4^2 \times 3.098^2 + 2 \times 0.6 \times 0.4 \times 12}$$

$$\sigma p = 3.56.$$

OR

$$\sigma p = \sqrt{W_x^2 \sigma x^2 + W_y^2 \sigma y^2 + 2 W_x W_y \, r_{xy} \sigma x \sigma y}$$

$$\sigma p = \sqrt{0.6^2 \times 3.87^2 + 0.4^2 \times 3.098^2 + 2 \times 0.6 \times 0.4 \times 1 \times 3.87 \times 3.098}$$

$$\sigma p = 3.56$$

(VIII) % of Risk diversification

$$= \frac{\text{Weighted } \sigma p - \text{Actual } \sigma p}{\text{Weighted } \sigma p} \times 100\%$$

$$= \frac{3.56 - 3.56}{3.56} \times 100\% = \underline{\underline{0\%}}$$