CONCEPT OF TIME VALUE OF MONEY.

This concept tries to explain why individuals prefers to receive cash today rather than receiving the same amount in the future. As time passes, the value of money will change and thus purchasing power declines as time passes due to inflation.

Individuals prefers current cash than the future cash due to the following reasons.

1. Because of the availability of the investment opportunities
2. In order to take advantage of the cash discount being offered.
3. Because of future uncertainty of cashflows
4. Because of the individuals subjective preference for money i.e individuals prefers cash for their own reasons.

The concept is analyzed as follows.

1. Future value (FV)

a) Future value of a single Amount

The process of determining the future value of a given amount is known as Compounding.

→ The general formula is as follows

$$FV = PV(1+r)^n$$

Illustration 1

Consider an investor who have invested sh 100,000 in a bank earning an interest rate of 10% p.a. Determine the future value at the end of the 3rd year.

Solution:

Period	Balance bfd	Interest 10%	Balance cfd
1	100,000	10,000	110,000
2	110,000	11,000	121,000
3	121,000	12100	133 100

OR

$$FV = PV(1+r)^n$$

$$100,000(1+0.1)^3 = 133,100$$

b) Future value of an Annuity.

An annuity refers to the equal amount received or paid after equal period eg salary, insurance premium, rent etc.

→ There are 2 types of annuities

(i) Ordinary Annuity (End) (arrears)

This is where the cashflow occurs at the end of each period.

(ii) Annuity due (Advance)

This is where the cashflow occurs at the beggining of the period eg rent, insurance premium etc.

illustrations 1

Consider an individual who expects to receive sh 10,000 at the end of each year for the next 3 years. If the prevailing interest rate is 10%, determine the future value of the amount

Formular = FV of annuity = $\text{Annuity}\left[\frac{(1+r)^n - 1}{r}\right]$

soln

$$10{,}000 \times \left[\frac{(1+0.1)^3 - 1}{0.1}\right] = \text{Ksh } 33{,}100$$

illustration 2

Now Assume that the individual above was to receive the Ksh 10,000 at the beggining of the year. Determine the future value of the annuity due.

solution:

$$FVA = \text{Annuity}\left[\frac{(1+r)^n - 1}{r}\right](1+r)$$

$$= 10{,}000\left[\frac{(1+0.1)^3 - 1}{0.1}\right](1+0.1) = \text{Ksh } 36{,}410$$

OR

Period	$FV = PV(1+r)^n$	FV
1	$10{,}000(1+0.1)^3$	13,310
2	$10{,}000(1+0.1)^2$	12,100
3	$10{,}000(1+0.1)^1$	11,000
		36410

2. PRESENT VALUE (PV)

This involves determining the present value of a future value. This process is known as discounting

a) present value of single amount

$PV = FV(1+r)^{-n}$

where: $(1+r)^{-n}$ → is the present value interest factor (pvif) written as follows $PVIF^{n}_{r\%}$

$PV = FV \times PVIFA^{n}_{r\%}$

Illustrations 1

Assume you expect to receive Ksh 1000000, 4 years from now. If the prevailing interest rate is 12%, determine the present value of the amount.

solution

$PV = FV(1+r)^{-n}$

$1000000\,(1+0.12)^{-4} =$ Ksh 635,518

Illustration 2

Consider a 3 year project which is expected to generate the following cashflows:

Year	1	2	3
Cashflows	80000	60000	50000

If the discounting rate is 10%, determine the present value of the cashflows

solution:

Period	Cashflow	$PVIF^{n}_{10\%}$	PV
1	80000	$(1+0.1)^{-1} = 0.9091$	72728
2	60000	$(1+0.1)^{-2} = 0.8264$	49584
3	50000	$(1+0.1)^{-3} = 0.7513$	37566.
			159,878

b) present value of Annuity

It has the following formulae

$$PVA = A\left[\frac{1-(1+r)^{-n}}{r}\right]$$

when the cashflow happens at the beggining of the period.

$$PVA = A\left[\frac{1-(1+r)^{-n}}{r}\right](1+r)$$

c) present value of annuity to perpetuity or infinity

Is determined as follows

$$PVA\infty = \frac{A}{r}$$

(d) **Present value of a growing annuity till Infinity**

This is where the annuity is expected to increase at a constant rate until infinity.

$$PVA_A^{\infty} = \frac{\text{Annuity in period 1}}{\text{Cost of Capital} - \text{Constant growth rate}} = \frac{A_1}{r-g}$$

(e) **Present value of differential Annuities**

These are equal cashflows that occurs in between economic life of the project. The eg Assuming an annuity of ksh 50,000 p.a between year 4 to year 10, determine the present value.

$$= \text{Annuity}\left(PVIFA_r^{10} - PVIFA_r^{3}\right)$$

May 2018 Q 4 d.

Mawa ltd is in the process of completing construction of a green house. The finance manager has estimated that the project's useful life is 15 years and shall generate the following cashflows.

Years	Cashflows "sh000"
1—5	5000
6—10	9000
11—15	4000
	18000

The required rate of return for the company is 10%.

Required:

Total present value of the project (5 marks)

Solution.

Year.	Cashflows	$PVIF_r^n$.		sh'000" present value
1—5.	5000	$PVIFA_{10\%}^{5}$ =	3.7908	18,954
6—10	9000	$PVIFA_{10\%}^{10} - PVIFA_{10\%}^{5}$	(6.1446 − 3.7908) = 2.3538	21,184.2
11—15	4000	$PVIFA_{10\%}^{15} - PVIFA_{10\%}^{10}$	(7.6061 − 6.1446) = 1.4615	5 846.
				45,984.2

illustration 2

Consider a project expected to generate the following cashflows

Year	1—4	5—9	10—∝
Cashflows	50,000	90,000	40,000

If the discounting rate is 10%. Determine the present value

Solution.

period.	Cashflows	$PVIFA_r^n$.		Present value
1—4	50,000	$PVIFA_{10\%}^{4}$	3.1699	158495
5—9	90,000	$PVIFA_{10\%}^{9} - PVIFA_{10\%}^{4}$	(5.7590 − 3.1699) = 2.5891	233,019
10—∝	40,000	$PVA\infty - PVIFA_{10\%}^{9}$	$\left(\frac{1}{0.1} - 5.7590\right)$ = 4.2410	169640
				561,154

LOAN AMORTIZATION SCHEDULE

This is a schedule showing the interest expense/charge and the principle amount borrowed.

→ the installment payment represents the annuity while the amount borrowed today will represent the present value of an annuity

$$\text{Installment payment} = \frac{\text{Amount Borrowed}}{PVIFA_{r\%}^{\overline{n}}}$$

The installment payment consists of 2 element

a) Interest charges
b) principle amount.

Illustrations

Nov 2011 Q3a

Makika Gulyo borrowed Ksh 4000,000 from Uhuluma Bank at an annual compound interest of 14% on a reducing balance. The loan was payable in annual installment over a period of four years. The installment were payable at the end of the year.

Required: A loan amortization schedule (6mks)

Solution

$$\text{Installment payment} = \frac{\text{Amount Borrowed}}{PVIFA_{14\%}^{4}} = \frac{1000,000}{2.9137} = 343,206$$

Amortization schedule

Period	Balance (A) bfd.	Interest (B) charge 14%	payment C	Principle D = C − B	Balance cfd. E = A − D
1	1000,000	140,000	343206	203 206	796 794
2	796 794	111551	343206	231 655	565 139
3	565139	79119	343 206	264087	301052
4	301052	42147	343 206	301 052	0

Dec 2006 Q 4b

Ben Kareeta obtained a loan of Ksh 4,000,000 on 1 Jan 2005. The rate of interest was fixed at 12% per annum. The loan is to be repaid semi-annually over a period of 4 years. Prepare a loan amortization schedule over the 4 year period.

Solution:

$$\text{Installment payment} = \frac{4000,000}{PVIFA_{6\%}^{8}} = \frac{4000,000}{6.2098} = 644,143$$

Amortization schedule

Period	Balance bcd (A)	interest (B) charge 6%	payment (C)	principle D=(C-B)	Balance cd A-D
1 - 1	4000000	240000	644143	404143	3595857
- 2	3595857	215751	644143	428393	3167464
2 - 3	3167464	190048	644143	454095	2713369
- 4	2713369	162802	644143	481341	2232028
3 - 5	2232028	133922	644143	510221	1721807
- 6	1721807	103308	644143	540835	1180972
4 - 7	1180972	70858	644143	573285	607687
- 8	607687	36461	644143	607682	0

Assignment: Dec 2013 Q 2C
Dec 2007 Q3C

LEASES

Types of leases

1. operating lease
2. Capital / Finance lease
3. sell and lease Back lease
4. Leverage lease

} Read more from Handout